# TEMPERATURE GRADIENT AT THE CONICAL END OF A SEMI-INFINITE CYLINDRICAL ROD

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UDC 536.24.02

A procedure is given for calculating the temperature gradient at the end of a cylindrical rod terminating in a truncated cone, with allowance for radiation and variations of the physical characteristics.

A number of problems (such as analysis of the energy balance of the cathode spot in high-pressure arcs) require knowledge of the temperature gradient at the end of an electrode. Several papers have been published on this problem [1, 2].

We investigate heat transfer in a semi-infinite cylindrical rod whose end is in the shape of a truncated cone. We consider the one-dimensional problem. Approximate calculations have shown that the main factor governing the balance of power of the electrode beyond the limits of the cathode spot in arcs carrying currents up to 20 or 30 A and operating at pressures up to 10 atm is thermal radiation from the electrode. The action of all other factors such as discharge radiation, heat convection and conduction through the gas, and heating by the passage of current has virtually no effect on the thermal balance. Consequently, the heat-conduction equation can be limited to the effect of cooling by thermal radiation (which cannot be neglected, because the resulting error would be excessive). Heating of the electrode by thermal radiation from surrounding bodies, mainly the walls enclosing the discharge space, can be neglected by virtue of their much lower temperature in comparison with the electrode temperature ($2T_{walls} < T$ [2]) and the small value of the wall emissivity (the walls are usually glass or quartz). Heating of the electrode by its own reflected radiation is also negligible.

We can thus reduce the problem to the solution of the nonlinear differential equation

$$\frac{d}{dX}\left(\lambda\pi r^2\frac{dT}{dX}\right)=\frac{2\pi rP_{\mathrm{r}}'}{\cos\gamma}=\frac{2\pi r\sigma\varepsilon}{\cos\gamma}T^4 \tag{1}$$

subject to the boundary (end) conditions

$$X=0,\quad T=T_{\mathrm{e}};\quad X\to\infty,\ \frac{dT}{dX}\to 0. \tag{2}$$

It may be assumed for many metals and alloys that

$$\lambda(T)=\lambda_0(1+CT), \tag{3}$$

$$\varepsilon(T)=-\varepsilon_0(T^2+a_1T+a_2), \tag{4}$$

in good agreement with the data of [3, 4].

Equation (1) with the boundary conditions (2) can be represented in the equivalent form

$$-\lambda\frac{dT}{dX}=\frac{2}{r^2(X)}\sqrt{r_1^3\int_0^{T_1}\frac{\lambda P_{\mathrm{r}}'}{\cos\gamma}\,dT+\int_{T_1}^{T(x)}\frac{r^3[X(T)]\,\lambda P_{\mathrm{r}}'}{\cos\gamma}\,dT}, \tag{5}$$

which yields the required value of the temperature gradient at any point of the rod.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 28, No. 2, pp. 268-271, February, 1975. Original article submitted October 9, 1973.

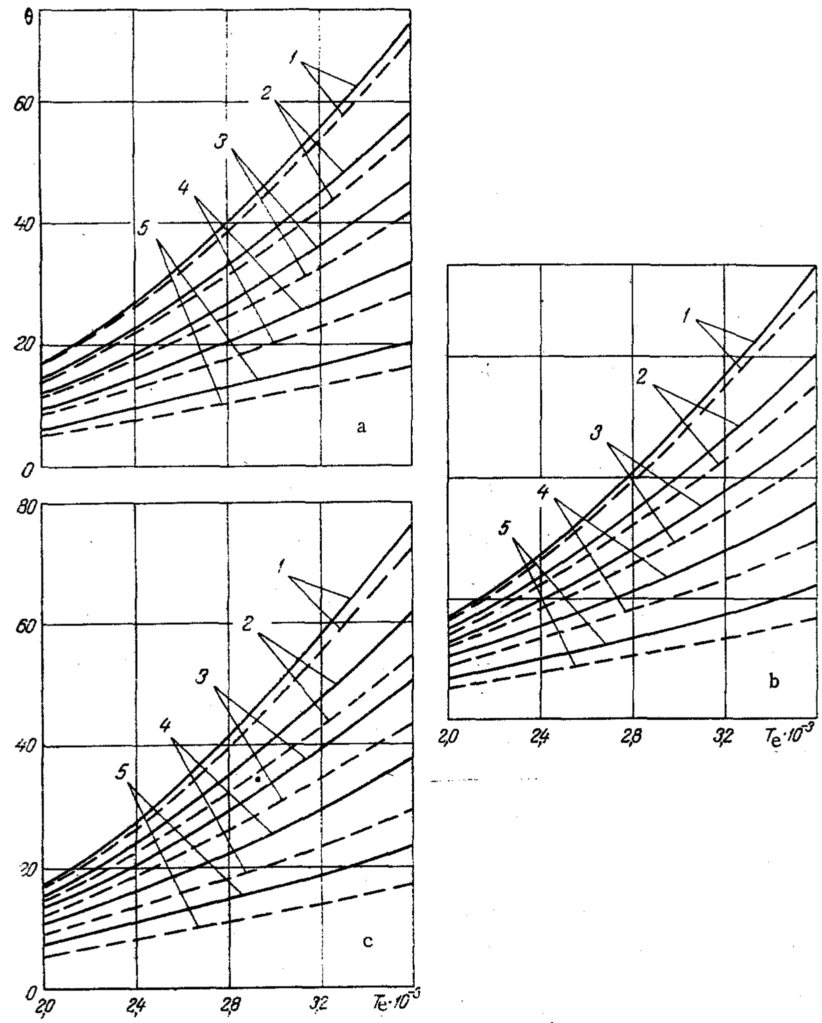


Fig. 1. Characteristic $\theta$ ($W \cdot m^{-3/2}$) versus temperature $T_e$ (°K) at end of the rod for R = 5 (solid curves) and R = 25 (dashed curves) and for various values of $\psi = (\sqrt{r_1/\cos\gamma}) \cdot [(R-1)/\tan\gamma]$: 1) $\psi = 0.05\ m^{1/2}$; 2) 0.125; 3) 0.25; 4) 0.50; 5) 1.25; a) $\gamma = 30°$; b) 45°; c) 60°.

The first integral on the right-hand side of (5) is readily evaluated. The second integral can only be determined numerically, requiring exceedingly difficult calculations in the determination of X(T) and $T_1$; r(x) is specified by the rod geometry.

The values of $T_1$ and the second integral in (5) can be determined by successive approximations. It is assumed in the first approximation that heat losses from the conical section of the rod are absent. We then obtain in the second approximation

$$-\lambda \frac{dT}{dX} = \frac{y^2}{\sqrt{r_1}} \sqrt{\varphi(T_1, \gamma = 0) + W(T, \gamma)}, \tag{6}$$

where

$$W(T, \gamma) = \frac{a^4}{2(R-1)} \left\{ a^2\left(1 - \frac{1}{y^2}\right) + 12ab\left(1 - \frac{1}{y}\right) + 30b^2 \ln y \right\} [\zeta(T_e, \gamma) - \zeta(T_1, \gamma)]; \tag{7}$$

$$\varphi(T, \gamma) = -\frac{4\sigma\varepsilon_0\lambda_0 T^6}{\cos\gamma} \left[ \frac{1}{8} CT^2 + \frac{1}{7}(1 + a_1 C) T + \frac{1}{6}(a_1 + a_2 C) + \frac{1}{5} \cdot \frac{a_2}{T} \right]; \tag{8}$$

$$b = \frac{T_e - T_1}{R-1}; \quad a = T_1 - b; \tag{9}$$

$$\zeta(T, \gamma) = -\frac{4\sigma\varepsilon_0\lambda_0}{\cos\gamma}\left[\frac{1}{2}CT^2 + (1 + a_1C)T + (a_1 + a_2C)\ln T - \frac{a_2}{T}\right]; \tag{10}$$

$$T_1 = \sqrt[3]{-q + \sqrt{p^3 + q^2}} + \sqrt[3]{-q - \sqrt{p^3 + q^2}} - \frac{\lambda_0 C}{6\rho}; \tag{11}$$

$$\rho = \frac{\sqrt{r_1}}{\operatorname{tg}\gamma}(R-1)\sqrt{\frac{\varphi(T_e, \gamma)}{T_e^6}}; \tag{12}$$

$$2q = -\frac{\lambda_0(1 + 0.5CT_e)T_e}{\rho} - \frac{\lambda_0^2 C}{6\rho^2} + 2\left(\frac{\lambda_0 C}{6\rho}\right)^3; \tag{13}$$

$$3p = \frac{3\lambda_0\rho - 0.25\lambda_0^2 C^2}{3\rho^2}. \tag{14}$$

The temperature dependence of $\theta = -\lambda(dT/dX)|_{T=T_e}(\sqrt{r_1}/R^2)$ for a tungsten rod ($\lambda_0 = 124\ W\cdot m^{-1}$ $(°K)^{-2}$, $C = -8.06\cdot10^{-5}\ (°K)^{-1}$ [5]; $\varepsilon_0 = 4.05\cdot10^{-8}\ (°K)^{-2}$, $a_1 = -7.05\cdot10^3$ °K, $a_2 = 3.58\cdot10^6\ (°K)^2$ [6]) is shown in Fig. 1 for several combinations of geometric parameters of the rod.

A comparison of the analytical result obtained according to Eqs. (6)-(14) with the numerical solutions shows that for a tungsten rod with variations of $T_e$ from 1600 to 3600°K, and of $\sqrt{r_1}(R-1)\cot\gamma$ from 0.04 to 1.0 $m^{1/2}$, $R \geq 5$, the error in the determination of $\lambda(dT/dX)|_{T=T_e}$ does not exceed 3%, i.e., the calculations can be limited to the second approximation.

## NOTATION

| | |
|---|---|
| $\lambda$ | is the thermal conductivity; |
| $\varepsilon$ | is the total emissivity; |
| $r$ | is the radius of the rod at any point; |
| $T$ | is the absolute temperature; |
| $X$ | is the lengthwise coordinate; |
| $P_r'$ | is the specific radiation density; |
| $\sigma$ | is the Stefan–Boltzman constant; |

$y = r_1/r$; $R = r_1/r_e$; $\tan\gamma = [(r - r_e)/X](X < X_1)$; $\gamma = 0(X > X_1)$.

### Subscripts

| | |
|---|---|
| 1 | is the boundary between the cylindrical and conical sections of the rod; |
| e | is the end of the rod (smaller base of the truncated cone). |

## LITERATURE CITED


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